

“Long” Quantum Superstrings in $AdS_5 \times S^5$

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Abstract

We discuss the computation of quantum corrections near long IIB superstring configurations in $AdS_5 \times S^5$ space which are related to the Wilson loop expectation values in the strong coupling expansion of the dual $\mathcal{N} = 4$ SYM theory with large N . We use the Green-Schwarz description of superstrings in curved R-R backgrounds and demonstrate that it is well-defined and useful for developing perturbation theory near long string background.

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1 Introduction

Further progress in clarifying the relation between strings and large N (non) supersymmetric gauge theories depends on better understanding of superstrings in AdS -type spaces with Ramond-Ramond (RR) fluxes [1, 2]. The simplest most symmetric example is provided by $AdS_5 \times S^5$ background of type IIB superstring theory. One could hope that this theory, though quite nontrivial being dual to large N $\mathcal{N} = 4$ Super Yang-Mills theory [2], should be, like the flat superstring, explicitly solvable.

To be able to treat this theory in an efficient way one is to use the Green-Schwarz (GS) approach [3] where one views the string as moving in a superspace (x^m, θ_α^I) with θ 's being space-time spinors. Then the coupling of the string to RR background has a local form $\bar{\theta}\Gamma\cdots\theta\partial x\partial x F\cdots$, just like its coupling to the curvature. It is useful to note that while the string “feels” the metric already at the classical level, it interacts with the RR background only through its quantum fluctuations.

There seems to be no alternative to the GS-type description: even the simplest one-loop computation in GS formalism like the one described in [4] would require a summation of infinite number of diagrams with any number of flat-space RR vertex operator insertions if done in NSR formalism. In fact, the use of GS action is very natural from “solitonic” point of view. If one views fundamental strings as “electric”-type objects appearing in $D = 10$ supergravity, the effective action for collective coordinates of a long such string will naturally have a GS form, as dictated by the residual space-time (super)symmetries (the characteristic non-linear $\partial x\bar{\theta}\Gamma\cdots\theta$ kinetic term originates from the standard Dirac $D = 10$ fermion action reduced to the world surface of the string, with Dirac matrices becoming the projected ones). The theory should of course contain not only long but also short strings which should also be described by the same manifestly space-time supersymmetric action.

A simple way to construct the flat space GS action is to view strings as propagating in flat coset superspace $= (10\text{-d super Poincare group})/(10\text{-d Lorentz group})$. The action is then a kind of Wess-Zumino-Witten action which has global supersymmetry and local κ -symmetry which ensures the correct number of the fermionic degrees of freedom. It is defined in terms of the basic objects which are the left-invariant Cartan 1-forms on the type IIB coset superspace $G^{-1}dG = L^{\hat{A}}P_{\hat{A}} + L^I Q_I$, $L^{\hat{A}} = dX^M L_M^{\hat{A}}$, $X^M = (x, \theta)$, where $[P_{\hat{A}}, P_{\hat{B}}] = 0$, $\{Q_I, Q_J\} = -2i\delta_{IJ}(\mathcal{C}\Gamma^{\hat{A}})P_{\hat{A}}$ and $G = G(x, \theta)$ is a coset representative $G(x, \theta) = \exp(x^{\hat{A}}P_{\hat{A}} + \theta^I Q_I)$. The coset space vielbeins are given by $L^{\hat{A}} = dx^{\hat{A}} - i\bar{\theta}^I \Gamma^{\hat{A}} d\theta^I$, $L^I = d\theta^I$. Here $x^{\hat{A}}$ are flat bosonic coordinates and θ^I ($I = 1, 2$) are two left Majorana-Weyl 10-d spinors. The general form of the action is [5]

$$S = -\frac{1}{2} \int_{\partial M_3} d^2\sigma \sqrt{g} g^{\mu\nu} L_\mu^{\hat{A}} L_\nu^{\hat{A}} + i \int_{M_3} \mathcal{H}, \quad \mathcal{H} = s^{IJ} L^{\hat{A}} \wedge (\bar{L}^I \Gamma^{\hat{A}} \wedge L^J), \quad (1)$$

where $s^{IJ} \equiv \text{diag}(1, -1)$, $2\pi\alpha' = 1$ and $d\mathcal{H} = 0$. The 2-d metric $g_{\mu\nu}$ ($\mu, \nu =$

0, 1) has signature $(-, +)$, and $g \equiv -\det g_{\mu\nu}$. The first “kinetic” term in the action has “degenerate” metric, containing only the square of translational but not spinor Cartan forms, as the latter would lead to an action quadratic in fermionic derivatives and thus to potential non-unitarity. The explicit 2-d component form of the GS action is [3]

$$S_0 = \int d^2\sigma \left[-\frac{1}{2}\sqrt{g}g^{\mu\nu}(\partial_\mu x^{\hat{A}} - i\bar{\theta}^I \Gamma^{\hat{A}} \partial_\mu \theta^I)(\partial_\nu x^{\hat{A}} - i\bar{\theta}^J \Gamma^{\hat{A}} \partial_\nu \theta^J) \right. \\ \left. - i\epsilon^{\mu\nu} s^{IJ} \bar{\theta}^I \Gamma^{\hat{A}} \partial_\nu \theta^J (\partial_\mu x^{\hat{A}} - \frac{1}{2}i\bar{\theta}^K \Gamma^{\hat{A}} \partial_\mu \theta^K) \right]. \quad (2)$$

The key observation that allows one to construct the GS action for superstrings in $AdS_5 \times S^5$ is that this bosonic space is a coset

$$G/H = SO(2, 4)/SO(1, 4) \times SO(6)/SO(5)$$

and also has maximal supersymmetry. As a result, one can consider strings moving not in flat superspace but in supercoset space with the right bosonic part and right number supersymmetries: one is to replace the 10-d super Poincare group by a smaller one $PSU(2, 2|4)$ and the Lorentz group $SO(1, 9)$ by its subgroup $SO(1, 4) \times SO(5)$, i.e. to consider the supercoset $PSU(2, 2|4)/[SO(1, 4) \times SO(5)]$ [6], which still has 10 bosonic and 32 Grassmann dimensions.

The bosonic part of the GS action is simply the standard symmetric space sigma model $L = \text{Tr}[(g^{-1}\partial g)_{G/H}]^2$. This is obviously not a conformal sigma model; it is the addition of the fermions θ that converts it into a conformal 2-d model of a novel type. The fermions couple the AdS_5 and S^5 parts of the action together and contribute the RR 5-form term to the beta-function of the metric making it vanish [6], $R_{mn} - (F_5 F_5)_{mn} = 0$.

The superalgebra $psu(2, 2|4)$ plays the central role in the construction of the GS action in $AdS_5 \times S^5$ [6]. Its even part is the sum of the algebra $so(4, 2)$ which is the isometry algebra of AdS_5 and the algebra $so(6)$ which is the isometry algebra of S^5 . The odd part consists of 32 supercharges corresponding to 32 Killing spinors in $AdS_5 \times S^5$ vacuum [7] of type IIB supergravity [8]. In the “5+5” split its generators are: $(P_A, J_{AB}; P_{A'}, J_{A'B'}; Q_{\alpha\alpha'I})$, i.e. the two sets of translations and rotations in $so(2, 4)$ and $so(6)$ ($A = 0, 1, \dots, 4; A' = 1, \dots, 5$) and 32 supercharges ($\alpha = 1, 2, 3, 4; \alpha' = 1, 2, 3, 4$). The commutation relations of $psu(2, 2|4)$ in $so(4, 1) \oplus so(5)$ basis are [6] $[P_A, P_B] = J_{AB}$, $[P_{A'}, P_{B'}] = -J_{A'B'}$, $[J^{AB}, J^{CE}] = \eta^{BC} J^{AE} + \dots$, $[J^{A'B'}, J^{C'E'}] = \eta^{B'C'} J^{A'E'} + \dots$, $[Q_I, P_A] = -\frac{i}{2}\epsilon_{IJ} Q_J \gamma_A$, $[Q_I, J_{AB}] = -\frac{1}{2}Q_I \gamma_{AB}$, $[Q_I, P_{A'}] = \frac{1}{2}\epsilon_{IJ} Q_J \gamma_{A'}$, $[Q_I, J_{A'B'}] = -\frac{1}{2}Q_I \gamma_{A'B'}$, and

$$\{Q_{\alpha\alpha'I}, Q_{\beta\beta'J}\} = \delta_{IJ}[-2iC_{\alpha'\beta'}(C\gamma^A)_{\alpha\beta}P_A + 2C_{\alpha\beta}(C'\gamma^{A'})_{\alpha'\beta'}P_{A'}] \\ + \epsilon_{IJ}[C_{\alpha'\beta'}(C\gamma^{AB})_{\alpha\beta}J_{AB} - C_{\alpha\beta}(C'\gamma^{A'B'})_{\alpha'\beta'}J_{A'B'}], \quad (3)$$

where C, C' are charge conjugation matrices. Then $G^{-1}dG = L^A P_A + \frac{1}{2}\hat{L}^{AB} \hat{J}^{AB} + L^{A'} P_{A'} + \frac{1}{2}L^{A'B'} J^{A'B'} + L^{I\alpha\alpha'} Q_{I\alpha\alpha'}$, and the coset representative can be parametrised as $G(x, \theta) = g(x) \exp(\theta \cdot Q)$.

Following the same steps as above in the construction of the flat space GS action one finds [6] that there exists a unique action of the type (1) which has the required properties: its bosonic part is sigma model on $AdS_5 \times S^5$, it has local κ -symmetry and global $PSU(2, 2|4)$ symmetry, and it reduces to the standard GS action in the flat space limit. The action has the same structure as (1) but now in terms of the 1-forms corresponding to the supercoset $PSU(2, 2|4)/[SO(1, 4) \times SO(5)]$. The leading terms in the resulting action are essentially the covariantisation of the flat space action with the ordinary derivatives on θ 's replaced by generalized $AdS_5 \times S^5$ covariant ones (containing extra terms linear in Dirac matrices that correspond to the interaction with RR background).

As in the standard WZW model case it is possible to argue that this action defines a conformal sigma model [6, 16]: (i) the WZ term in the action is not renormalized since the divergences are local and manifestly covariant, while the kinetic L^2 term in the action can be renormalized only by an overall constant because of the global symmetries of the sigma model; (iii) assuming that κ -symmetry is preserved by regularization, it should relate the coefficients of the kinetic and the WZ terms as it does at the classical level (i.e. κ -symmetry plays the role of the affine symmetry in WZW model).

The explicit form of all higher order terms in θ in the action of [6] can be found in [9, 10]. Fixing a “covariant” κ -symmetry gauge (e.g. $\theta^1 = \Gamma_{0123}\theta^2$) one gets a relatively simple action which contains terms of quadratic and quartic order in fermions only [11, 12]. The resulting action has the same feature as the flat space GS action: its fermionic kinetic term is coupled to derivative of all bosonic string coordinates. It is non-degenerate when expanded near “long” string configuration [13], and thus the GS action provides a well-defined and useful tool for computing quantum corrections to “long” string configurations ending on Wilson loops [14] at the boundary of AdS_5 [13, 15, 4].

The bosonic part of the action for a string in $AdS_5 \times S^5$ is the sigma model corresponding to the space-time metric ($m = 1, \dots, 4$)

$$ds^2 = R^2 \left[\frac{1}{w^2} (dw^2 + dx^m dx^m) + d\Omega_5^2 \right]. \quad (4)$$

The α' expansion corresponds to the large 't Hooft coupling λ expansion in the dual gauge theory, with the leading classical contribution proportional to $\frac{R^2}{\alpha'} = \sqrt{\lambda}$ [14].

2 Quadratic approximation

The part of the action in [6] quadratic in θ^I is a direct generalization of the quadratic term in the flat-space GS action (2)

$$S_F^{(2)} = i \int d^2\sigma (\sqrt{g} g^{\mu\nu} \delta^{IJ} - \epsilon^{\mu\nu} s^{IJ}) \bar{\theta}^I \rho_\mu D_\nu \theta^J. \quad (5)$$

Here ρ_μ are projections of the 10-d Dirac matrices,

$$\rho_\mu \equiv \Gamma_{\hat{m}} E_M^{\hat{m}} \partial_\mu x^M = (\Gamma_A E_M^A + \Gamma_{A'} E_M^{A'}) \partial_\mu x^M, \quad (6)$$

and $E_M^{\hat{m}}$ is the vielbein of the 10-d target space metric. The covariant derivative D_μ is the projection of the 10-d derivative $D_M = \partial_M + \frac{1}{4} \omega_M^{\hat{m}\hat{n}} \Gamma_{\hat{m}\hat{n}} - \frac{1}{8 \cdot 5!} \Gamma^{\hat{m}_1 \dots \hat{m}_5} \Gamma_M e^\Phi F_{\hat{m}_1 \dots \hat{m}_5}$ which appears in the Killing spinor equation of type IIB supergravity,

$$D_\mu \theta^I \equiv \left(\delta^{IJ} D_\mu - \frac{i}{2} \epsilon^{IJ} \tilde{\rho}_\mu \right) \theta^J, \quad \tilde{\rho}_\mu \equiv \left(\Gamma_A E_M^A + i \Gamma_{A'} E_M^{A'} \right) \partial_\mu x^M, \quad (7)$$

where $D_\mu = \partial_\mu + \frac{1}{4} \partial_\mu x^M \omega_M^{\hat{m}\hat{n}} \Gamma_{\hat{m}\hat{n}}$ and the term with $\tilde{\rho}_\mu$ originates from the coupling to the RR 5-form field strength (note that F_5 is proportional to the ϵ -tensors on $AdS_5 \times S^5$). The presence of the generalized ‘‘Killing spinor’’ covariant derivative explains why the action has 32 global supersymmetries. Fixing the natural type IIB κ -symmetry gauge $\theta^1 = \theta^2 \equiv \theta$ one finds from (5)

$$S_F^{(2)} = 2i \int d^2 \sigma \left(\sqrt{g} g^{\mu\nu} \bar{\theta} \rho_\mu D_\nu \theta - \frac{i}{2} \epsilon^{\mu\nu} \bar{\theta} \rho_\mu \tilde{\rho}_\nu \theta \right). \quad (8)$$

This action gives a well-defined fermion kinetic term if one expands near generic ‘‘long’’ string configuration. It is easy to check directly that the bosonic $AdS_5 \times S^5$ sigma model becomes indeed 1-loop finite when supplemented by the fermionic action (8) [4] with the ‘‘mass term’’ in (8) originating from the RR coupling playing the crucial role.

The kinetic term in GS action in curved target space background (8) involving θ 's which are world-volume scalars can be transformed into the ‘‘2-d spinor’’ form, i.e. into the standard kinetic term for a set of 2-d Dirac fermions defined on a curved 2-d space. In the flat space case expanding the GS action near $x^0 = \tau$, $x^1 = \sigma$ one concludes that (e.g. in the $\theta^1 = \theta^2$ gauge) the kinetic fermionic term takes the form $\bar{\theta} \Gamma^a \partial_a \theta$, $a = 0, 1$, so that choosing the representation of 10-d Dirac matrices where $\Gamma^{0,1}$ are proportional to 2-d Dirac matrices one may re-interpret the action as the one for a collection of 2-d Dirac fermions.

In general, to achieve a similar transformation one is to apply a local target space Lorentz rotation to GS θ (as discussed previously mostly in flat space in [17]). Identifying the 2-d metric $g_{\mu\nu}$ with the induced metric $G_{mn} \partial_\mu x^m \partial_\nu x^n$ we can write the first covariant derivative term in (8) as $i \int d^2 \sigma \sqrt{g} g^{\mu\nu} (\bar{\theta} \rho_\mu \partial_\nu \theta - \partial_\mu \bar{\theta} \rho_\nu \theta)$. Introducing the tangent t_α^m ($m = 0, 1, \dots, 9$, $\alpha = 0, 1$) and normal n_s^m ($s = 1, \dots, 8$) vectors to the world surface which form orthonormal 10-d basis ($g_{\mu\nu} = e_\mu^\alpha e_\nu^\beta \eta_{\alpha\beta}$), i.e. $t_\alpha^m = e_\alpha^\mu \partial_\mu \bar{x}^m$, $(t_\alpha, t_\beta) = \eta_{\alpha\beta}$, $(t_\alpha, n_s) = 0$, $(n_s, n_u) = \delta_{su}$, where $(a, b) = G_{mn} a^m b^n$, one can make a local $SO(1, 9)$ rotation of this basis which transforms the set of σ -dependent 10-d Dirac matrices into the 10 constant Dirac matrices $\rho_\alpha(\sigma) = e_\alpha^\mu \rho_\mu = S(\sigma) \Gamma_\alpha S^{-1}(\sigma)$, $\rho_s(\sigma) = n_s^m E_m^{\hat{a}} \Gamma_{\hat{a}} = S(\sigma) \Gamma_s S^{-1}(\sigma)$. One may further choose a representation in which $\Gamma_\alpha = \tau_\alpha \times I_8$, where τ_α are 2-d Dirac matrices.

Depending on specific embedding and particular curved target space metric, one may then be able to write the action (5) as the action for 2-d Dirac fermions coupled to curved induced 2-d metric and interacting with some 2-d gauge fields (coming from $S^{-1}dS$).

Simple examples of when this happens [4] will be discussed below. We shall consider embeddings of the string world sheet into the AdS_3 part of the AdS_5 space, so that there will be only one normal direction and the extra normal bundle 2-d gauge connection will be absent.

3 One-loop partition function for “straight” string

The simplest classical solution for string in $AdS_5 \times S^5$ is a straight string with the world surface spanned by the radial direction of AdS_5 and time. The Euclidean solution and the corresponding induced metric are

$$x^0 = \tau, \quad x^4 \equiv w = \sigma, \quad ds^2 = \frac{1}{\sigma^2}(d\tau^2 + d\sigma^2). \quad (9)$$

The induced metric on the world sheet is that of AdS_2 , with constant negative curvature $R^{(2)} = -2$ (we set the radius R of AdS_5 to 1). This solution corresponds to a single straight Wilson line at the boundary running along the Euclidean time direction. This is a BPS object in string theory (and boundary gauge theory): it corresponds to a static fundamental string stretched between a single D3-brane (placed at the boundary of AdS_5) and N coinciding D3-branes (placed at the horizon and supporting $AdS_5 \times S^5$ by their RR flux). Therefore, the partition function should be equal to 1. The properly defined (subtracted) classical string action evaluated on this background vanishes, and the corresponding 1-loop correction to the vacuum energy defined with respect to a certain time-like Killing vector vanishes too (see below). Relating the vacuum energy to the partition function using a conformal rescaling argument one concludes that $Z = 1$ [4].¹ The calculation of the partition function is rather subtle, and depends on a regularization prescription and proper inclusion of measure factors. For practical applications, the precise value of Z (which is simply a constant) is not actually important, and one may normalize with respect to it in computing Z for more general string configurations. Any smooth Wilson loop looks in the UV region like a straight line. This translates into the behavior of the minimal surface near the boundary of AdS_5 space. In the general case one will have to calculate the partition function for a complicated two dimensional field theory. But asymptotically the minimal surface will approach AdS_2 , and

¹It should be mentioned that while the properly defined vacuum energy of a supersymmetric field theory in AdS space should vanish, this does not automatically imply (in contrast to what happens in flat space) that the partition function of such theory should be equal to 1 (cf. [18, 19]).

the small fluctuation operators (in particular, the asymptotic values of the masses of the fluctuation fields) will also be the same as for a straight string. Subtleties related to divergences and asymptotic boundary conditions can be automatically avoided in more general cases by normalizing with respect to the partition function of the straight string.

The bosonic part of the action for small fluctuations in conformal gauge turns out to be (a, b are the tangent indices of AdS_5 part and p of S^5 part)

$$S_{2B} = \frac{1}{2} \int d^2\sigma \sqrt{g} \left(D^\mu \zeta^a D_\mu \zeta^a + X_{ab} \zeta^a \zeta^b + D^\mu \zeta^p D_\mu \zeta^p \right), \quad (10)$$

where $X_{ab} = \text{diag}(1, 2, 2, 2, 1)$. The only non-trivial components of the co-variant derivative are $D_0 \zeta^0 = \partial_0 \zeta^0 - w^{-1} \zeta^4$, $D_0 \zeta^4 = \partial_0 \zeta^4 + w^{-1} \zeta^0$. Because of the direct embedding of the world sheet into the target space the projection of the target space connection on the world sheet is the same as the spin connection of the induced metric and thus the action of the conformal gauge ghosts is identical to the action of the longitudinal modes ζ^0, ζ^4 .

The quadratic part of the fermion action in the $\theta^1 = \theta^2$ gauge (8) is

$$L_{2F} = -2i\sqrt{g}(\bar{\theta}\rho^\mu \hat{\nabla}_\mu \theta + i\bar{\theta}\rho_3\theta), \quad \rho_3 \equiv \frac{1}{2}\epsilon^{\alpha\beta}\rho_\alpha\rho_\beta = \Gamma_{04}, \quad (11)$$

where $\rho_\alpha = (\Gamma_0, \Gamma_4)$ may be identified with (2-d Dirac matrices) $\times I_8$. Thus the quadratic fermionic part of GS action has exactly the same form as the action for 2-d fermions in curved AdS_2 space with a mass term. Assuming the standard $\int d^2\sigma \sqrt{g} \bar{\theta}\theta$ normalization, the corresponding Dirac operator is $i\rho^\mu \hat{\nabla}_\mu - \rho_3 = iw(-\Gamma_0\partial_0 + \Gamma_4\partial_1) - \frac{1}{2}i\Gamma_4 - \Gamma_0\Gamma_4$ and its square is $-\hat{\nabla}^2 + \frac{1}{4}R^{(2)} + 1$.

Ignoring the ghosts and longitudinal modes² we are left with a 2-d field theory on AdS_2 containing 5 massless scalars, 3 scalars with $(\text{mass})^2 = 2$, and eight fermions with $(\text{mass})^2 = 1$. Supersymmetric field theories on AdS_2 were studied, e.g., in [20, 18, 21]. The fields in the $\mathcal{N} = 1$ scalar supermultiplet in AdS_2 may have the following bosonic and fermionic masses [22, 18, 23]: $m_B^2 = \mu^2 - \mu$, $m_F = \mu$, (μ is a free parameter). We thus have 5 “massless” multiplets with $\mu = 1$ ($m_B^2 = 0$, $m_F = 1$) and 3 “massive” multiplets with $\mu = -1$ ($m_B^2 = 2$, $m_F = -1$). It is possible to combine a $\mu = 1$ and a $\mu = -1$ multiplet into an $\mathcal{N} = 2$ multiplet (the dimensional reduction of the 4-d chiral multiplet to 2 dimensions). Two $\mu = 1$ multiplets also form an $\mathcal{N} = 2$ multiplet (a dimensional reduction of the 4-d vector multiplet). Three chiral and one vector multiplets in $D = 4$ make one $\mathcal{N} = 4$ vector in four dimensions, and that suggests that the 8 scalars and 8 fermions that we have should form one $\mathcal{N} = 8$ multiplet in two dimensions (cf. [24]). We finally obtain the following partition function for the transverse and fermionic modes with the scalar and spinor Laplace operators defined with respect to

² Their contributions cancel each other modulo a constant related to difference in boundary conditions which is crucial for making the full string partition function finite [4].

the Euclidean AdS_2 metric with radius 1 ($R^{(2)} = -2$)

$$Z_{B+F} = \frac{\det^{8/2} \left(-\hat{\nabla}^2 + \frac{1}{4} R^{(2)} + 1 \right)}{\det^{3/2} (-\nabla^2 + 2) \det^{5/2} (-\nabla^2)} . \quad (12)$$

One should impose proper boundary conditions consistent with AdS_n supersymmetry [25]. Those imply that the resulting spectra of Laplace operators are discrete in spatial direction (and not continuous as one would expect in a non-compact hyperbolic space).

Instead of calculating the partition function directly, we may start with the vacuum energy. It is given by the determinant of the operator scaled to remove the factor of g^{00} from in front of ∂_t^2 (see [19]). Changing the world sheet coordinates so that the AdS_2 metric becomes $ds^2 = \cos^{-2} \rho (dt^2 + d\rho^2)$, $\rho \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and using the spectra of the Hamiltonians conjugate to the time variable t calculated in [26] $\omega_n^{(F)}(\mu) = n + |\mu| + \frac{1}{2}$, $\omega_n^{(B)}(\mu) = n + h(\mu)$, $h(\mu) = \frac{1}{2}(1 + \sqrt{1 + 4m_B^2})$, one finds, summing over all the modes as in [23], the 1-loop vacuum energy of this effective 2-d field theory

$$E = \frac{1}{2} \sum_{n=0}^{\infty} \left(3 \left[\omega_n^{(B)}(-1) - \omega_n^{(F)}(-1) \right] + 5 \left[\omega_n^{(B)}(1) - \omega_n^{(F)}(1) \right] \right) . \quad (13)$$

The *properly defined* vacuum energy should *vanish* in the AdS case as it does in flat space [20, 26, 18, 27] (even though divergences may not cancel out, unless there is a lot of supersymmetry [28]). The direct computation of the sum of the mode energies using ζ -function regularization gives (using the standard relations $\zeta(s, x) \equiv \sum_{n=0}^{\infty} (n+x)^{-s}$, $\zeta(-1, x) = -\frac{1}{2} \left(x^2 - x + \frac{1}{6} \right)$)

$$E = -\frac{1}{4} \left[3 \times \left(2 + \frac{1}{6} \right) + 5 \times \frac{1}{6} - 8 \times \left(1 - \frac{1}{12} \right) \right] = 0 . \quad (14)$$

The ratio 3:5 of the numbers of the two multiplets is just what is needed for the cancellation. The fact that E , defined by ζ -function regularization, vanishes should be a consequence of the extended $\mathcal{N} = 8$ supersymmetry mentioned above. Indeed, while as was found for AdS_4 [20, 18, 27], the ζ -function regularization may break supersymmetry and thus may lead to $E \neq 0$, this does not actually happen in the case of $N \geq 5$, $d = 4$ gauged supergravities [28]. The present $d = 2$ case is thus analogous to those $d = 4$ cases with large amounts of supersymmetry.

In curved (e.g., static conformally flat) space the logarithm of the partition function is, in general, different from the vacuum energy defined as a sum over eigen-modes because the time derivative part of the relevant elliptic operators is rescaled by g^{00} . The determinants of the two operators which differ by such a rescaling are related to each other by a conformal anomaly type equation. Taking this into account one finally concludes [4] that $Z_{tot} = 1$. This result is a consequence of $E = 0$ as well as of the cancellation of the sum of conformal anomalies for ten bosons with total mass terms 8, eight

fermions with 4 times the standard 2-d fermion conformal anomaly and total mass 8, and the conformal gauge ghosts.

Similar results are found in the case of the string world sheet ending on a circular Wilson loop at the boundary, where the induced geometry is again AdS_2 [4].

4 One-loop correction to “bended” string or rectangular Wilson loop

Analogous calculation of the one-loop correction can be done in the case of the “bended” string configuration with both ends being at the boundary of AdS_5 separated by distance L in a spatial direction, and representing the rectangular (T, L) Wilson loop in the boundary gauge theory [14].

The corresponding minimal surface was found in [14] and the classical value of the string action (equal to the minimal area as fermions vanish at the classical level) accounts for the leading large ‘t Hooft coupling $\lambda = \frac{R^4}{\alpha'^2}$ behavior ($\frac{c_0\sqrt{\lambda}}{L} = \frac{c_0 R^2}{\alpha' L}$) of the “quark – anti-quark” (W-boson) potential in the large N $\mathcal{N} = 4$ SYM theory. On general grounds, in this conformal field theory the Wilson loop should be $\langle W(C) \rangle \sim \exp[-TV(L)]$, $V(L) = -\frac{f(\lambda)}{L}$, where $f \rightarrow f_0 = d_1\lambda + d_2\lambda^2 + \dots$ at weak coupling and AdS/CFT duality predicts [14] that $f \rightarrow f_\infty = c_0\sqrt{\lambda} + c_1 + O(\frac{1}{\sqrt{\lambda}})$ at strong coupling. It is natural to expect that there is a smooth function $f(\lambda)$ which interpolates between the two expansions. The first correction $\frac{c_1}{L}$ to the strong-coupling expansion of the potential is determined by the one-loop (α') correction to the string partition function (see also [29] for related discussions).

The use of the GS action of [6] as a starting point for this one-loop calculation was suggested in [13] (where the fermionic contribution was found in the gauge $\theta^1 = \Gamma_{0123}\theta^2$) and it was completed in [15, 4].

Writing the $AdS_5 \times S^5$ metric ($R = 1$) as $ds^2 = y^2 dx^n dx^n + \frac{dy^2}{y^2} + d\Omega_5^2$ where $y \equiv x^4 = w^{-1}$, the string configuration is described by

$$x^0 = \tau \in (0, T), \quad x^1 = \sigma \in (-\frac{L}{2}, \frac{L}{2}), \quad y = y(\sigma), \quad (15)$$

where $y(\sigma)$ extremises the action [14] $S = \frac{R^2}{2\pi\alpha'} T \int d\sigma \sqrt{y'^2 + y^4}$, i.e. is determined by the second-order equation $yy'' = 4y'^2 + 2y^4$ with the first integral being $y'^2 = \frac{y^8}{y_0^4} - y^4$, $y_0 = y_{min} = \frac{\kappa_0}{L}$, $\kappa_0 \equiv \frac{(2\pi)^{3/2}}{[\Gamma(\frac{1}{4})]^2}$. The induced 2-d metric takes the form $ds^2 = y^2 d\tau^2 + y^6 d\sigma^2$, $R^{(2)} = -2(1 + y^{-4})$. The induced geometry is asymptotic to AdS_2 : if we change the coordinate σ to y the metric becomes

$$ds^2 = y^2 d\tau^2 + \frac{y^2}{y^4 - y_0^4} dy^2, \quad y_0 \leq y < \infty. \quad (16)$$

The $y_0 = 0$ limit corresponds to the straight string configuration where the metric becomes that of Euclidean AdS_2 space (with $0 \leq y < \infty$).

In the flat space limit ($R \rightarrow \infty$) one finds that the quantum correction to the rectangular Wilson loop vanishes because of the cancellation of the bosonic and fermionic contributions due to effective 2-d supersymmetry present after gauge fixing. The 1-loop $\frac{c_1}{L}$ correction to the effective potential should not, however, vanish [13] in the present curved space case as there is no reason to expect that the action expanded near the solution $y = y(\sigma)$ should have an effective world-sheet supersymmetry.

The bosonic fluctuations near this bended string configuration were analysed in [15, 4]. As in the straight string case the one-loop bosonic partition function in the static gauge is again expressed in terms of massive and massless scalar determinants in induced 2-d geometry. In the static gauge [15]

$$Z_B^{(stat.)} = \det^{-2/2}(-\nabla^2 + 2) \det^{-1/2}(-\nabla^2 + R^{(2)} + 4) \det^{-5/2}(-\nabla^2), \quad (17)$$

while in the conformal gauge [4]

$$Z_B^{(conf.)} = \frac{\det^{1/2}(-\nabla_{\mu\nu}^2 - \frac{1}{2}R^{(2)}g_{\mu\nu})}{\det^{1/2}(-D_{ab}^2 + X_{ab}) \det^{5/2}(-\nabla^2)}, \quad (18)$$

where $X_{ab} = 2\delta_{ab} - g^{\mu\nu}\eta_\mu^a\eta_\nu^b$, $\eta_0^a = (y, 0, 0, 0, 0)$, $\eta_1^a = (0, y, 0, 0, y^{-1}y')$ (we set $y_0 = 1$). The two expressions are equivalent once supplemented with constant factors related to gauge fixing.

As discussed above, a systematic way to put the fermionic contribution into that of massive 2-d Dirac spinors in induced 2-d geometry is to perform a local Lorentz rotation. The quadratic part of the fermionic action depends on $\rho_0 = y\Gamma_0$, $\rho_1 = y\Gamma_1 + y^{-1}y'\Gamma_4$, $\hat{D}_\mu = \partial_\mu + \frac{1}{2}y\Gamma_\mu\Gamma_4$. Using the gauge $\theta^1 = \theta^2$ (and Minkowski signature) one observes [4] that the combination of Γ_1 and Γ_4 which appears in ρ_1 can be interpreted as a (local, σ -dependent) rotation of Γ_1 in the 1-4 plane $S\Gamma_1S^{-1} = \cos\alpha\Gamma_1 + \sin\alpha\Gamma_4 = y^{-2}\Gamma_1 + y^{-4}y'\Gamma_4 = y^{-3}\rho_1$, where $S = \exp(-\frac{\alpha}{2}\Gamma_1\Gamma_4)$, $\cos\alpha = y^{-2}$, $\alpha' \equiv \frac{d\alpha}{d\sigma} = 2y$. Making the field redefinition $\theta \rightarrow \Psi \equiv S^{-1}\theta$, one obtains

$$L_{2F} = 2i\sqrt{g} \left(\bar{\Psi}\tau^\mu\hat{\nabla}_\mu\Psi + i\bar{\Psi}\tau_3\Psi \right), \quad \tau_3 \equiv \frac{\epsilon^{\mu\nu}}{2\sqrt{g}}\tau_\mu\tau_\nu = \Gamma_0\Gamma_1, \quad (\tau_3)^2 = 1. \quad (19)$$

Choosing a representation for Γ_a such that $\Gamma_{0,1}$ are \sim 2-d Dirac matrices, i.e. $\Gamma_0 = i\sigma_2 \times I_8$, $\Gamma_1 = \sigma_1 \times I_8$, $\tau_3 = \Gamma_0\Gamma_1 = \sigma_3 \times I_8$, we end up with 8 species of 2-d Majorana fermions living on a curved 2-d surface with a σ_3 mass term. Assuming that fermions are normalized with \sqrt{g} , the square of the resulting fermionic operator is then $-\hat{\nabla}^2 + \frac{1}{4}R^{(2)} + 1$, where $\hat{\nabla}^2 = \frac{1}{\sqrt{g}}\hat{\nabla}^\mu(\sqrt{g}\hat{\nabla}_\mu)$ and $\hat{\nabla}$ is the covariant 2-d spinor derivative of the induced metric. Equivalent results for the fermionic contribution were found in the “3-brane” gauge $\theta^1 = i\Gamma_4\theta^2$ [13] and in a different κ -symmetry gauge in [15].

Combining the contributions of bosons in the static gauge and fermions in the $\theta^1 = \theta^2$ gauge the total expression for the 1-loop partition function of

a “bended” string in $AdS_5 \times S^5$ is thus [15, 4]

$$Z = Z_0 \frac{\det^{8/2}(-\hat{\nabla}^2 + \frac{1}{4}R^{(2)} + 1)}{\det^{2/2}(-\nabla^2 + 2) \det^{1/2}(-\nabla^2 + R^{(2)} + 4) \det^{5/2}(-\nabla^2)}, \quad (20)$$

where Z_0 is a gauge fixing (measure) factor which is the same as in the flat ($R \rightarrow \infty$) limit. In particular, this factor is crucial for cancellation [4] of the topological ($\log \epsilon \int R^{(2)}$) divergence present [15] in the ratio of the determinants. The key point is that the cancellation of this divergence in the one-loop approximation in curved target space is essentially the same as in the flat space, where to demonstrate that logarithmic Euler number divergences cancel (as they should to match the cancellation of conformal anomalies in $D = 10$ superstring) one is to take into account various zero mode normalization factors in the string path integral measure [30].³ To avoid questions about boundary terms (and details of topological infinity cancellation) one may normalize the partition function for each field by the partition function of an equivalent field in the straight string configuration, i.e. divide the partition function for the noncompact hyperbolic negative curvature 2-d space (16) by the partition function for the AdS_2 case.⁴ Since the indexed 2-d geometry is static, the calculation of the partition function and thus of the one-loop coefficient c_1 in the potential $V(L)$ reduces to determining the spectra of one-dimensional Schrödinger operators, a tractable problem. A crude estimate for the resulting coefficient c_1 was discussed in [4].

5 Conclusions

To conclude, the Green-Schwarz action for the type IIB superstrings in $AdS_5 \times S^5$ background provides a natural and well-defined starting point for developing α' perturbation theory near “long” string configurations. The resulting partition function can be interpreted as that of an effective 2-d quantum field theory in curved induced 2-d geometry. The one-loop string

³One should note also that in performing a local target space rotation that transforms the quadratic GS fermion term into the 2-d fermion kinetic term the resulting Jacobian [31] depends on the 2-d metric and its contribution explains why the conformal anomaly of a GS fermion is 4 times bigger than that of a 2-d fermion. This is crucial for understanding how conformal anomalies cancel in $D = 10$ GS string.

⁴Since the topology and the near-boundary (large y) behavior of the two metrics is the same, this eliminates the problem of carefully tracking down all boundary terms in the expressions for the determinants and allows one to ignore the boundary contributions as well as the total derivative bulk terms (such as the logarithmically divergent terms proportional to $\int d^2\sigma \sqrt{g} R^{(2)}$). The ratio of the determinants for the metric (16) and for its $y_0 = 0$ limit will be finite and well-defined. This is the standard recipe of defining the determinants of Laplace operators on (e.g. 2-dimensional) non-compact spaces by using fiducial metrics of constant negative curvature which have the same topology and asymptotic behavior.

correction is already sensitive to details of the string coupling to RR background and would be hard to reproduce in the standard NSR formalism where the RR vertex operator is defined near flat space. One may hope that progress towards understanding the spectrum of “short” closed strings in $AdS_5 \times S^5$ and thus large N superconformal YM theory for any value of λ may be achieved using a light-cone gauge approach like the one initiated in [32].

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